Supplementary Material for Adaptive Exploration: What You See Is Up to You (Chapter 7 of *Taming Uncertainty)*

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When people make decisions from experience, they explore the options by sampling possible outcomes from each alternative. According to the Choice from Accumulated Samples of Experience model (CHASE), they subjectively evaluate these observations and accumulate these valuations, thereby forming a preference. They stop exploring based on either a predetermined level of preference or sample size and choose accordingly. We sketch the formal model here.

Search

In the standard sampling paradigm, participants decide how to explore by choosing one of the two options to generate an outcome. We summarize the search strategy in terms of the probabilities of sampling from each option, s_H and $s_L = 1 - s_H$. A simple strategy is that people divide their exploration equally between the two options ($s_H = s_L = .5$), which is consistent with the overall search behavior in the experiments described in Chapter 7. However, it is straightforward to model other strategies where search is guided by other factors like a particular option or the variance of the lottery.

Subjective evaluation

Each outcome is mapped to a utility of the outcome. We use a similar function from cumulative prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992; Wakker, 2010) to parameterize the function

$$\begin{aligned} u(x) &= x^{\alpha}, x > 0 \\ u(x) &= -\lambda |x|^{\alpha}, x \leq 0. \end{aligned} \tag{S7.1}$$

The parameter α determines the curvature of the utility function for both gains and losses, while λ determines the degree of loss aversion.

Accumulation

The outcome observed on draw t gives rise to a subjective valence, $v(t)$:

$$\begin{aligned} v(t) &= \omega[x(t)] \times u[x(t)], \text{if } H \text{ is sampled}, \\ v(t) &= -\omega[x(t)] \times u[x(t)], \text{if } L \text{ is sampled}. \end{aligned} \tag{S7.2}$$

Positive valence indicates that an outcome was evaluated as favoring option H; negative valence indicates favorability of option L. The term $\omega[x(t)]$ represents the weight given to each sampled outcome. We specify these weights next. The valence $v(t)$ then updates an evolving preference state P, which is a discrete-time, continuous-state stochastic process:

$$P(t) = P(t-1) + v(t). \tag{S7.3}$$

Attention weights

Typically, we might assume that each outcome receives an equal weight in the accumulation process, $\omega[x(t)] = 1$. However, we allow the weight an outcome receives to vary based on its likelihood and its favorability relative to the other possible outcomes in the gamble. This is accomplished by making the weight for each sampled outcome a function of its (de)cumulative rank in the gamble for gains, $r(x) = P(X \geq x)$, and cumulative rank in the gamble for losses, $r(x) = P(X \leq x)$. This assumption is equivalent to the one made to determine probability weights in cumulative prospect theory. In fact, we use the same nonlinear weighting function W to determine the sample weights so that

$$\omega(x) = \frac{W[r(x)] - W[r(y)]}{r(x) - r(y)}, \tag{S7.4}$$

where y is the other outcome from the gamble such that $y > x$ and for y, $\omega(y) = W[r(y)]/r(y)$. The astute reader will note that $\omega(x)$ is approximately equal to the derivative of the probability weighting function. Figure S7.1 illustrates the properties of this function as well as the corresponding probability weighting function. It can be shown that preferences from CHASE will in the limit (as $t \to \infty$) mimic preferences from rank-dependent expected utility models like cumulative prospect theory (Pleskac et al., 2019; Zeigenfuse et al., 2014).

Starting point

The initial preference state $P(0)$ captures any bias people may have towards either option upon presentation of the choice options. The data we consider do not have any strong a priori reasons to expect a bias in the initial preference state. However, we allow for variability to enter the process here as well such as due to incidental trial-order effects. We model this variability by assuming the initial preference state is distributed according to a truncated Laplace distribution that is centered at 0, indicating no bias towards either option. The spread of the distribution is controlled by a free parameter τ that controls how peaked the distribution is over the center.

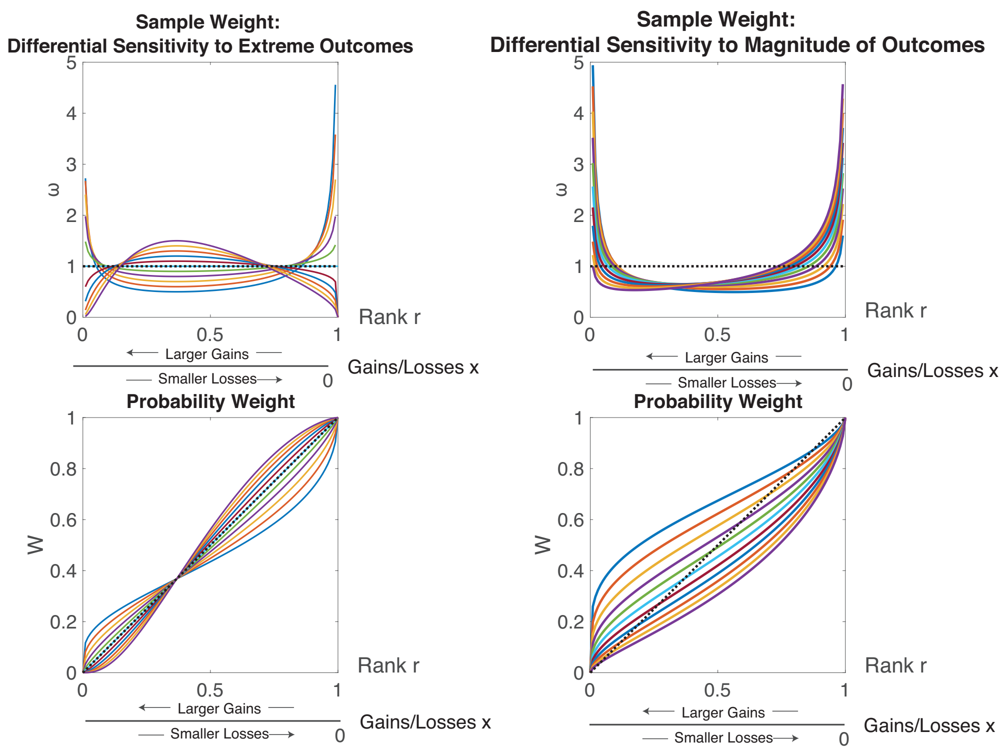

Figure S7.1. Properties of the sample weights and their relation to probability weights. The top row illustrates possible sample weights. Typically, each sample might be equally weighted with a value of 1. In CHASE, the sample weight of an outcome is a function of its decumulative (gains)/cumulative (losses) rank. The first horizontal axis denotes the rank r, and the second horizontal axis identifies the corresponding value x. The top left panel illustrates how the sample weights show differential sensitivity to either extreme or intermediate events. The top right panel illustrates how the sample weights show differential sensitivity to either high magnitudes or low magnitudes. The bottom row shows the corresponding probability weights such that in the limit choices with a given sample weight would appear to have the corresponding probability weighting function.

Stopping rule

Under optional stopping, a decision is made when the preference state passes a threshold for one of the choice options. This threshold is set by a respondent and determines the preference magnitude necessary to terminate sampling and choose an option. The H option is chosen when the preference passes the threshold θ so that

$$\begin{cases} \text{Choose } H \text{ if } P(s) \geq \theta. \\ \text{Choose } L \text{ if } P(s) \leq -\theta. \\ \text{Otherwise continue sampling.} \end{cases} \tag{S7.5}$$

References

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