Supplementary Material for The Robust Beauty of Heuristics in Choice Under Uncertainty (Chapter 2 of *Taming Uncertainty*)

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Outcome distributions

Each of the implemented environments represents a combination of a distribution of outcomes and a mechanism for the generation of the probabilities. Monetary gambles were sampled from each environment. We first describe the outcome distributions in more detail.

Five outcome distributions were implemented:

- A rectangular distribution, with values ranging between –500 and +500.
- A normal distribution, with a mean of 0 and a standard deviation of 250.
- An exponential distribution, with a mean of 100 and the values drawn being multiplied by –1 in half of the cases.
- A Cauchy distribution, the ratio of two values drawn from a standard normal distribution, multiplied by 10.
- A lognormal distribution, with $\mu = 0$ and $\sigma = 1$.

We selected these environments because they allowed us to vary the degree of skewness in monetary outcomes in the options, and thus to examine the impact of this environmental property on the performance of the choice heuristics. Any values above 25,000 or below –25,000 were set to these bounds. Figure S2.1 shows histograms that resulted from 1,000,000 draws from these distributions, illustrating the shape of the distributions. In addition, Table S2.1 reports descriptive statistics for samples drawn from these distributions. Note that these sampling distributions are not identical to the mathematical specifications of the parent distribution.

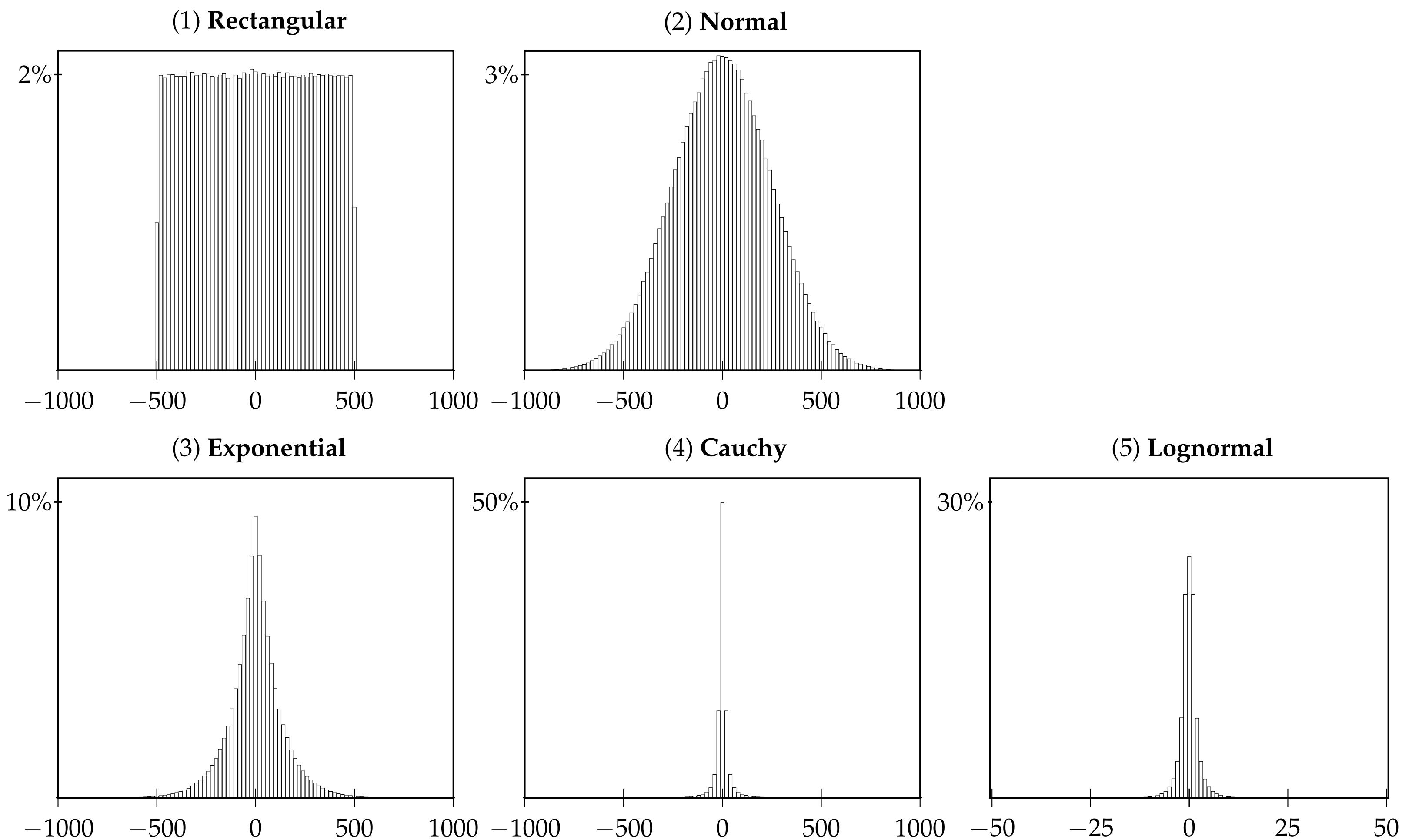

Figure S2.1. Histogram of 1,000,000 draws from the five outcome distributions used in the simulation. All distributions are symmetrical, but they differ in the degree of skewness (see Table S2.1). In the rectangular distribution, the mean and the median are the same.

Table S2.1: Mean, median, and standard deviation of 1,000,000 samples drawn from the five outcome distributions. Means and medians are based on absolute values (all distributions are symmetric around 0).

Distribution	Mean ($\|X\|$)	Median ($\|X\|$)	*SD*
Rectangular	250.1	250.0	288.9
Normal	199.7	168.9	250.2
Exponential	99.8	69.1	141.2
Cauchy	55.7	10.0	555.2
LogNormal	1.7	1.0	2.7

Construction principles for probabilities

The first three P-generators were determined using the following logic of construction. Any mechanism generating probabilities for the outcomes of each option needs to ensure that the probabilities within an option are additive. For each outcome within an option, a probability value was drawn from a distribution. The following three distributions were implemented:

- A rectangular distribution (over the interval $[0;1]$).
- A U-shaped beta distribution ($\alpha = \beta = 0.5$).
- An exponential distribution, in which the values were drawn from a rectangular distribution and then transformed by $f(x) = e^{10x}$.

Of course, the sum of the values drawn from the respective distributions will commonly not add up to 1. Therefore, all values for each of the three construction principles were divided by the sum of the values within each option. Due to this normalization, the expected size and dispersion of probabilities depends nontrivially on the number of outcomes. As the number of outcomes grows, it becomes increasingly unlikely that large probability values will be obtained for single outcomes. Therefore, we implemented a fourth P-generator, aiming to produce a substantial number of large probabilities, irrespective of the number of outcomes per option. To this end, the first probability value p_1 for k outcomes is determined by a random draw from the rectangular distribution over the interval $[0;1]$. No matter how many outcomes there are, this first draw will generate high values with the same probability. To accommodate a varying number of outcomes, we made the second draw dependent on the first: The value for the second outcome is drawn from the uniform interval $[0;1-p_1]$, so that the sum of the first two probabilities will always be less than or equal to 1. Following the same logic, the probability for outcome i is determined by a random draw from the interval $[0, 1-\sum_{j=1}^{i-1} p_j]$. The final probability value is determined to be $p_k = 1 - \sum_{j=1}^{k-1} p_j$ to ensure that all values add up to 1. Finally, the generated values are randomly assigned to the outcomes. The distribution of resulting probabilities stemming from this skewed generation mechanism and the other three mechanisms are plotted in Figure S2.2. Table S2.2 reports, separately for each generating mechanism, the Gini coefficient as a measure of the resulting variability in the probability values.

Table S2.2: Gini coefficients for the generated outcome probabilities.

	Mean (Gini)			Median (Gini)			SD (Gini)		
No. of outcomes	2	4	8	2	4	8	2	4	8
P-generator									
Rectangular	0.39	0.67	0.83	0.44	0.69	0.84	0.13	0.06	0.02
U-Shaped	0.32	0.62	0.81	0.38	0.65	0.82	0.18	0.11	0.04
Exponential	0.17	0.32	0.51	0.1	0.34	0.55	0.18	0.21	0.18
Skewed	0.33	0.56	0.7	0.37	0.63	0.77	0.15	0.18	0.18

Note. For each of the 12 cells, 1,000,0000 alternatives were generated. The table plots the mean coefficient, the median (of the median 1,000,000 coefficients), and the standard deviations. The Gini coefficient is calculated as 1 – sum of squares of the probabilities in a set. Smaller values indicate higher dispersion.

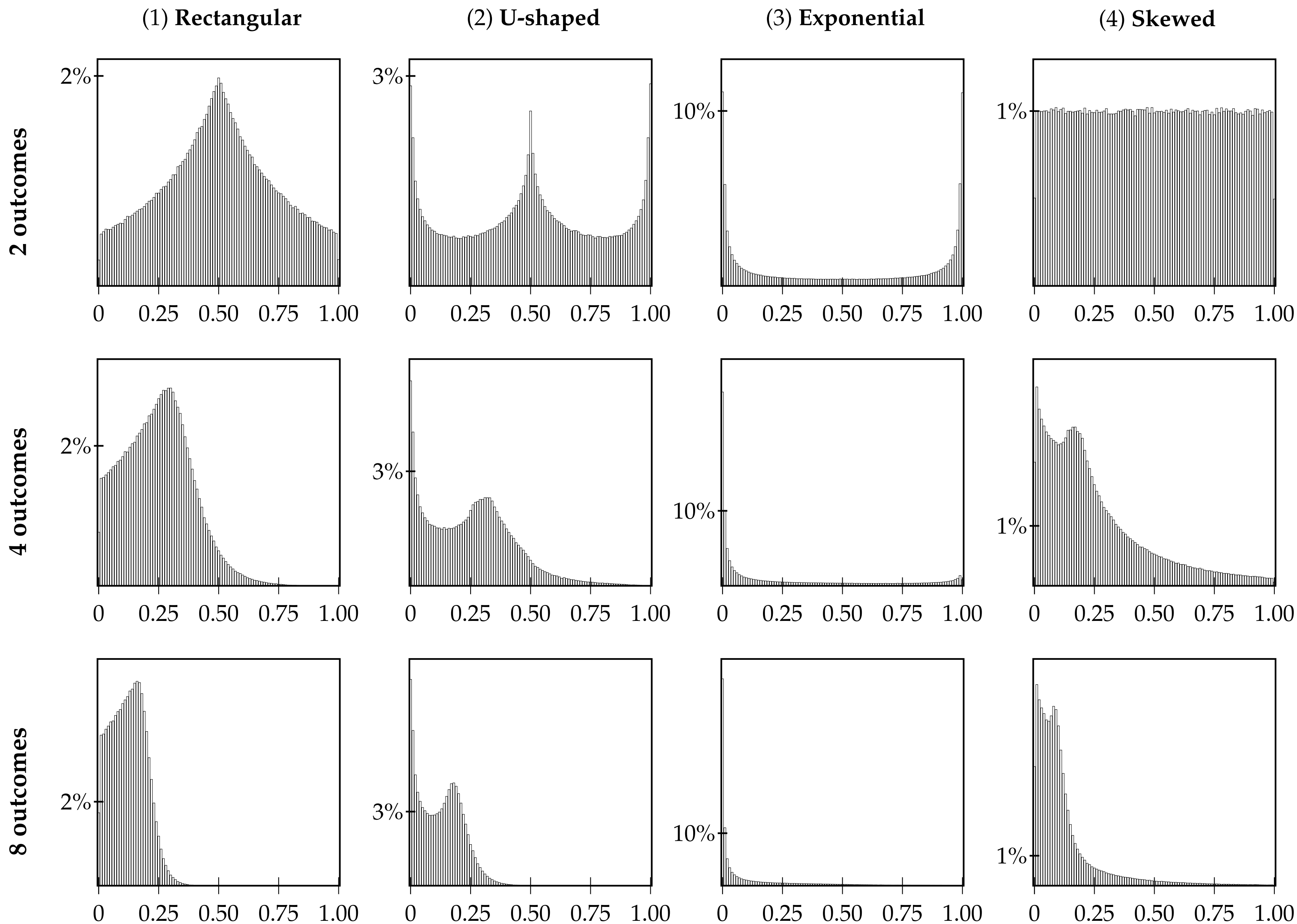

Figure S2.2. Examples of probability distributions based on 1,000,000 generated alternatives for each distribution and for two, four, or eight outcomes per alternative.